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VII. ON AN ARGUMENT FOR TRUTH-FUNCTIONALITY

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I. The Argument

A WELL-KNOWN argument of Quine's[1] is supposed to show that any context of a sentence admitting (without change of truth value) substitution of logically equivalent sentences and coextensive individual descriptions is a truth functional context of that sentence. The argument is as follows. Let 'p' represent any sentence, and let "$C(p)$" represent any sentence containing the sentence represented by 'p'. It is assumed that the sentence represented by "$C(p)$" retains its truth value if (1) the sentence represented by 'p' is replaced by any sentence logically equivalent to it, or if (2) any individual description in the contained sentence is replaced by a coextensive individual description. We notice that regardless of what sentence 'p' happens to represent, it is logically equivalent to

(i) $\hat{x}(x = \varphi \,\&\, p) = \hat{x}(x = \varphi)$.[2]

Hence, we may substitute (i) for 'p' in "$C(p)$" and obtain

(ii) $C(p) \equiv C(\hat{x}(x = \varphi \,\&\, p) = \hat{x}(x = \varphi))$.

If we assume

(iii) $p \equiv q$,

it follows that

(iv) $\hat{x}(x = \varphi \,\&\, p) = \hat{x}(x = \varphi \,\&\, q)$.

Now (iv) enables us to substitute "$\hat{x}(x = \varphi \,\&\, q)$" for "$\hat{x}(x = \varphi \,\&\, p)$" in (ii) yielding,

(v) $C(p) \equiv C(\hat{x}(x = \varphi \,\&\, q) = \hat{x}(x = \varphi))$.

But since whatever sentence 'q' represents, it is logically equivalent to

(vi) $\hat{x}(x = \varphi \,\&\, q) = \hat{x}(x = \varphi)$,

we may replace the occurrence of (vi) in (v) by 'q' obtaining

(vii) $C(p) \equiv C(q)$.

Since the only assumption made about the sentence represented by 'q' is (iii), it is concluded that the context in question is truth functional.

This argument is fallacious. It can be repaired in a way congenial to Quine's philosophical preferences, but at least one recent application of the argument cannot be salvaged in the repaired version.

II. The Fallacy

How is the set abstraction operator to be understood? There are two possibilities. (a) The set abstraction operator is to be defined contextually, perhaps as follows:

$$x \in \hat{x}Fx =_{df} Fx$$
$$\hat{x}Fx = \hat{x}Gx =_{df} (\forall x)\ (x \in \hat{x}Fx \equiv x \in \hat{x}Gx).$$

Combining these definitions we get,

$$\hat{x}Fx = \hat{x}Gx =_{df} (\forall x)\ (Fx \equiv Gx).$$

Now if we adopt this interpretation, the move from (iv) to (v) becomes illegitimate. For although (iv) is true, it does not assert the coextensiveness of two individual descriptions, but rather of two predicates, and we have no provision for the substitution of coextensive *predicates*. (b) The set abstraction operator makes definite descriptions out of predicates (open sentences). Now in first order logic with identity, "$\ldots c \ldots \supset (\exists x)\ (x = c)$" is provable, where '$c$' is any expression which can occupy the position of an individual variable. Thus, (i) logically implies the existence of $\hat{x}(x = \varphi)$ no matter what sentence 'p' represents. But then that sentence is not logically equivalent to the sentence represented by (i), since it is not the case that every sentence implies the existence of $\hat{x}(x = \varphi)$.

At this point it might be objected that every

[1] W. V. O. Quine, "Reference and Modality," in *From a Logical Point of View* (p. 159), and "Three Grades of Modal Involvement," in *Ways of Paradox*, (pp. 161–162). The argument ultimately goes back to Frege.

[2] 'ϕ' is here to be construed as a name for the empty set.

sentence (of a first order theory)[3] *does* imply the existence of $\hat{x}(x = \varphi)$ on the following grounds. First order theories contain axioms of identity (to express uniqueness, functionality, etc.). Given any individual constant 'c' of such a theory, we have the following proof:

$(\forall x)\ (x = x)$ (Axiom of identity)
Hence, $c = c$
Hence, $(\exists x)\ (x = c)$

In our case, we have "$(\exists x)\ (x = \hat{x}(x = \varphi)\ )$" as a theorem. If one regards identity theory as part of logic,[4] then this theorem is guaranteed by logic alone. So "$p \supset (\exists x)\ (x = \hat{x}(x = \varphi)$" will likewise be a theorem of logic.

Something has obviously gone wrong here: Must we say that "Grass is green," or even "There are no classes," logically imply the existence of $\hat{x}(x = \varphi)$? Surely this is going too far. For the objector's argument can be used to "logically guarantee" the existence of anything, since the following is also a proof.

$(\forall x)\ (x = x)$
Hence, Pegasus = Pegasus
Hence, $(\exists x)(x =$ Pegasus$)$

Obviously, the problem is that the adoption of a singular term as part of our vocabulary is not an ontologically neutral matter. To take an expression, e.g., the letter 'c', as a singular term in the vocabulary of a theory is to be committed to the existence of something named by 'c'. The axiom of identity, if counted as part of the logic of a first order theory at all, can only be done so in a first order theory without singular terms.

Still, a determined objector might argue as follows. "After all, the formula "$(\exists x)(x = c)$" is *valid*, assuming the ordinary interpretation of '='. So how can its appearance as a theorem of a first order theory be objectionable?"

The answer is that "$(\exists x)(x = c)$" is logically valid — comes out true in every interpretation — only as long as 'c' has the status of an uninterpreted individual constant. For then, by the definition of an interpretation, we must have a non-empty universe for the quantifier to range over, and 'c' must be assigned *some thing* in that universe, thus making "$(\exists x)(x = c)$" true. But if 'c' is a meaningful singular term, then "$(\exists x)(x = c)$" is no longer logically valid, for it asserts the existence of a thing named by that term.

"Well then, can't the argument be salvaged simply by rewriting it using the uninterpreted individual constant 'c' for 'φ'?" This question reveals a rather subtle misunderstanding of the role of the symbolic expressions in the argumentation of Sect. I. What Sect. I purports to provide is an argument schema. The *claim* made for this schema is that it is valid, i.e., that its instances are good arguments. An instance of a schema is the result of fixing the interpretation of all the uninterpreted symbols in some way or other which is consistent with the syntactic roles played by those symbols. In our case, since the interpretations of such symbols as "$\hat{x}(\quad)$," '=', and '&' are assumed fixed in the usual way, what is required is that 'p' and 'q' be interpreted as some particular sentences, "$C(\quad)$" as some particular containing sentence. Now if we replace 'φ' with 'c', we must ask what role 'c' is to play. If it is simply an uninterpreted symbol which is not a place-holder for a meaningful expression, then we no longer have a schema instances of which are arguments, but a mixture of prose on the one hand, and symbolic expressions of undetermined import on the other. On the other hand, if, as its syntactic position suggests, 'c' is treated as a place-holder for singular terms, we shall have an argument schema, but not a valid one. For it is not true that, regardless what sentence 'p' happens to represent, and what singular term 'c' happens to represent, 'p' and "$\hat{x}(x = c\ \&\ p) = \hat{x}(x = c)$" are logically equivalent. For example, "Grass is green," is not logically equivalent to "$\hat{x}(x = \varphi\ \&$ Grass is green$) = \hat{x}(x = \varphi)$." In fact, the suggestion that individual constants of the first order calculus be interpreted as place-holders for singular terms on a par with sentence letters results in the serious anomaly that instances of theorems of the first order calculus may be false. For example, '$(\exists x)(x =$ Pegasus$)$' is a false instance of the theorem "$(\exists x)(x = c)$."

We may see our way to reducing the gross anomaly of getting a falsehood as an instance of a theorem of logic by noticing that even with predicate letters, we must choose our English predicates carefully. "$(Fa\ \&\ a = b) \supset Fb$" is a provable

[3] By a first order theory, we mean a formal theory with a given interpretation whose underlying logic is standard first order logic. Cf. Alonzo Church, *Introduction to Mathematical Logic*, § 07 and 55. In particular, it is to be emphasized that the logic in question is not a "free logic," since, in such a logic, the following argument is clearly not valid.

[4] We do not wish to commit ourselves for or against such a position. If identity theory is *not* regarded as part of logic, then the objection under consideration is *clearly* without merit. Hence we argue under the opposite assumption.

schema, but if we let "John believes —— is tall" go in for 'F ——' we shall have well-known problems on our hands. The moral is that for the purposes of logic, an English predicate is not just a sentence minus some singular terms, but rather an expression that functions referentially as a predicate — i.e., is true or false of objects.[5] We may say similarly in the case of an individual constant that it can be replaced only by an expression which functions referentially as a singular term — i.e., denotes some object. Thus, "Something is Pegasus" will not be regarded as an instance of "$(\exists x)(x=c)$." But "Something is the sun" will be. And this is the lesser anomaly we are left with: Whereas "Something is the sun" is true, it is hardly logically true.

It emerges, then, that (i)–(vii) cannot be reformulated using an individual constant in place of 'φ'. For either the individual constant is not to be construed as a schematic letter, in which case Sect. I does not provide us with an argument at all, or the individual constant is to be treated as schematic for singular terms, in which case our original point remains: The sentence represented by 'p' will not be logically equivalent to that represented by (i), only the latter implying the existence of $\hat{x}(x = \varphi)$.

III. Repairs

Following the hint in (a) of the last section, let us assume that substitution of coextensive predicates in the contained sentence leaves the truth value of the containing sentence untouched. The desired conclusion will then be forthcoming. The repaired argument runs as follows. Regardless of what sentence 'p' happens to represent, it is logically equivalent to

(i) $(\forall x)(p \,\&\, x = x \equiv x = x)$.

Hence we may substitute (i) for 'p' in "$C(p)$" and obtain

(ii) $C(p) \equiv C(\,(\forall x)(p \,\&\, x = x \equiv x = x)\,)$.

If we assume

(iii) $p \equiv q$,

it follows that

(iv) $(\forall x)(p \,\&\, x = x \equiv q \,\&\, x = x)$.

Now (iv) allows us to replace "$p \,\&\, x = x$" by "$q \,\&\, x = x$" in (ii), yielding

(v) $C(p) \equiv C(\,(\forall x)(q \,\&\, x = x \equiv x = x)\,)$.

But since, whatever sentence 'q' represents, it is logically equivalent to

(vi) $(\forall x)\ (q \,\&\, x = x \equiv x = x)$,

we can replace the occurrence of (vi) in (v) by 'q' and obtain

(vii) $C(p) \equiv C(q)$.

This reformulation of the argument we take to be well within the spirit of Quine's often expressed preference for the elimination of singular terms in favor of predicates.

IV. Uses of the Argument

In an attempt to show that the logical form of singular causal sentences such as

(1) The short circuit caused the fire.

cannot be given by sentences such as

(2) *The fact that* there was a short circuit *caused it to be the case that* there was a fire.

where the italicized words constitute some sort of sentential connective, Donald Davidson employs a version of the argument under discussion.[6]

[5] The same point is made by Massey, *Understanding Symbolic Logic*, pp. 228ff.

[6] Donald Davidson, "Causal Relations," *Journal of Philosophy*, vol. 64 (1967), pp. 694–695. Davidson regards the argument on pages 197–198 of Quine's *Word and Object* as closely related to that of Sect. I. Briefly, Quine is there arguing against a version of modal logic which limits the universe over which the variables range to objects which are not specifiable in ways that fail of necessary equivalence. This limitation is to be accomplished by adopting a postulate, viz.

$$(\exists x)\ (Fx \,\&\, (\forall y)\ (Fy \supset y=x) \,\&\, Gx \,\&\, (\forall y)\ (Gy \supset y=x)\,) \supset \mathcal{N}(\forall x)\ (Fx \equiv Gx).$$

From this postulate Quine deduces that, for any sentence p, if p then $\mathcal{N}p$. The argument, slightly reformulated, is as follows: Assume p, and let 'a' and 'b' be any singular terms such that $a=b$. We then have

$a=b \,\&\, (\forall y)\ (y=b \supset y=a)$, and

$(a=b \,\&\, p) \,\&\, (\forall y)\ (\,(y=b \,\&\, p) \supset y=a)$.

From the postulate, with "Fx" as "$x=b$" and "Gx" as "$x=b \,\&\, p$," we then have

$\mathcal{N}(\forall x)\ (x=b \equiv (x=b \,\&\, p)\,)$.

Instantiating with respect to 'b' we get

$\mathcal{N}(b=b \equiv (b=b \,\&\, p)\,)$.

By standard modal logic we finish with

$\mathcal{N}(b=b)$, hence

$\mathcal{N}(b=b \,\&\, p)$, hence

$\mathcal{N}p$.

This argument is different from that of Sect. I. In particular, this argument, unlike that of Sect. I, is valid.

> It is obvious that the connective in (2) is not truth-functional, since (2) may change from true to false if the contained sentences are switched. Nevertheless, substitution of singular terms for others with the same extension in sentences like (1) and (2) does not touch their truth value.[7]

Davidson supports this claim with two examples, and then continues,

> Surely also we cannot change the truth value of the likes of (2) by substituting logically equivalent sentences for sentences in it. Thus, (2) retains its truth if for "there was a fire" we substitute the logically equivalent '$\hat{x}(x=x$ & there was a fire$)=\hat{x}(x=x)$'; retains it still if for the left side of this identity we write the coextensive singular term "$\hat{x}(x=x$ & Nero fiddled)"; and still retains it if we replace "$\hat{x}(x=x$ & Nero fiddled$)=\hat{x}(x=x)$" by the logically equivalent "Nero fiddled." Since the only aspect of "there was a fire" and "Nero fiddled" that matters to this chain of reasoning is the fact of their material equivalence, it appears that our assumed principles have led to the conclusion that the main connective of (2) is, contrary to what we supposed, truth-functional.[8]

The problem, of course, is that "there was a fire" is not logically equivalent to "$\hat{x}(x = x$ & there was a fire$) = \hat{x}(x = x)$," since the latter, but not the former, implies the existence of the universal set. This commitment to the universal set can be avoided if "$\hat{x}(x = x$ & there was a fire$) = \hat{x}(x = x)$" is construed as virtual notation asserting the coextensiveness of two predicates. But this move is of no help to Davidson: Although "substitution of singular terms for others with the same extension in sentences like (1) and (2) does not touch their truth value," substitution of general terms for others with the same extension in sentences like (1) and (2) *does* touch their truth value. To see this we need only reflect that whatever caused it to be the case that John F. Kennedy was born second son to Joseph and Rose Kennedy quite certainly did not cause it to be the case that John F. Kennedy was inaugurated President of the United States in 1960. Yet "was born second son to Joseph and Rose Kennedy" and "was inaugurated President of the United States in 1960" are coextensive predicates.

Davidson remarks that, "Having already seen that the connective of (2) cannot be truth-functional, it is tempting to try to escape the dilemma by tampering with the principle of substitution which led to it."[9] Davidson clearly regards such a strategy as unpromising and undesirable. We agree that tampering with substitutivity of identicals and of logical equivalents in the likes of (2) is undesirable. But these principles lead to no dilemma. Substitutivity of coextensive *predicates* and of logical equivalents in the likes of (2) does lead to a dilemma, but, in this case, tampering is not undesirable, it is mandatory. Substitution of coextensive predicates does not leave the truth-value of the likes of (2) untouched.

Quine makes use of an argument similar to the argument of Sect. I to show that the assumption of transparency for the context "Tom believes that . . ." is tantamount to the assumption of truth-functionality for that context.

> Where 'p' represents a sentence let us write 'δp' (following Kronecker) as short for the description:
>
> the number x such that (($x=1$) and p) or (($x=0$) and not p).
>
> We may suppose that Tom . . . is enough of a logician to believe a sentence of the form "$\delta p=1$" when and only when he believes the sentence represented by 'p'.[10]

Now suppose Tom believes that Cicero denounced Cataline. Then Tom believes that δ(Cicero denounced Cataline) $= 1$. But, assuming 'p' represents a true sentence, $\delta p = \delta$(Cicero denounced Cataline). Substituting "δp" for "δ(Cicero denounced Cataline)" in "Tom believes that δ (Cicero denounced Cataline)," which is justified by our assumption that the context "Tom believes that . . ." is transparent, we have that Tom believes that $\delta p = 1$. From this, and the assumption that Tom believes sentences of the form "$\delta p = 1$" when and only when he believes the sentence represented by 'p' we have that Tom believes that p.

This argument avoids the difficulties present in the version of Sect. I by requiring, not that 'p' and "$\delta p = 1$" are logically equivalent, but by assuming that Tom believes sentences of the form "$\delta p = 1$" when and only when he believes the sentence represented by 'p'. Now Quine *implies* that Tom does only what any good logician would do and with this we must disagree. For while sentences of the form "$\delta p = 1$" commit one to a number, the

[7] *Ibid*, p. 694.
[8] *Ibid*, p. 695.
[9] *Ibid*, p. 695.
[10] W. V. O. Quine, *Word and Object*, *op. cit.*, pp. 148–149.

sentence p need not, and in general, will not. But, although the rest of us, on reflection, may not believe sentences of the form "$\delta p = 1$" when and only when we believe the sentence p, it is quite possible that *Tom* does. Thus, though somewhat misleading, Quine's argument does establish the desired conclusion, viz., that the context, "x believes that . . ." cannot be generally treated as transparent.

V. Postscript

Having established the invalidity of Quine's argument for the thesis that the truth-functionality of a context is guaranteed by substutivity of logical equivalents and co-designative singular terms, there remains the question of its truth. Interestingly, what appears to be a counter-example to the thesis is suggested by Davidson's discussion of singular causal statements. Consider the following sentence.

> The fact that Smith fainted caused it to be the case that Jones ran for help.

The contained sentences ("Smith fainted" and "Jones ran for help") may always be replaced by sentences logically equivalent to them without changing the truth value of the entire sentence. Also, the singular terms in the contained sentences may always be replaced by other singular terms having the same referents without changing the truth value of the containing sentence.[11] Yet it is well-known that the context

The fact that . . . caused it to be the case that—.

is not truth functional.

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[11] Davidson gives some confirming examples in his discussion on pp. 694–695 of "Causal Relations," *Journal of Philosophy*, vol. 64, (1967).